

The University of Chicago

A METHOD FOR DETERMINING THE
ABSOLUTE AFFECTIVE VALUE
OF A SERIES OF STIMULUS
SITUATIONS

A PART OF A DISSERTATION SUBMITTED TO
THE GRADUATE FACULTY IN CANDIDACY FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF PSYCHOLOGY

By
AARON PAUL HORST

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METHOD FOR DETERMINING THE ABSOLUTE AFFECTIVE VALUE OF A SERIES OF STIMULUS SITUATIONS*

I. INTRODUCTION

The object of this investigation was to develop a technique whereby series of stimulus situations could be scaled with reference to their affective value. By "affective value" we mean simply the degree to which the stimulus situation evokes feelings of pleasantness or unpleasantness. The method assumes that there is a continuous gradation from those experiences which are extremely unpleasant to those which are extremely pleasant.

For example, let us imagine a scale as indicated in Fig. 1:

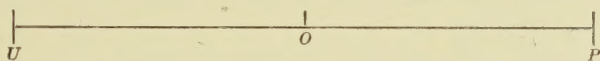


FIG. 1.

We assume that any stimulus situation may be allocated on this scale according to the degree of affect which it normally evokes in an individual. In the following analysis we attempt to measure the distance which any given situation has when allocated with reference to its affective value, from the point of zero affect. This distance is to be measured in terms of units to be subsequently defined.

The statistical technique involved is a modification and extension of a method developed by Thurstone.¹ The technique makes possible the determination of scale separations, on a psychological continuum of items in a series of dissimilar stimulus situations. So far as the method is concerned, these situations may be anything, providing they can be compared with reference to the degree of affect which they evoke.

Evidently any given stimulus situation will not evoke the same degree of affect in all individuals, nor will it evoke the same degree of affect in a given individual from one time to another. Without committing ourselves as to the manner in which an individual evaluates the affective value of a stimulus situation or distinguishes between the relative affective values of two or more situations, we may assume that

* Grateful acknowledgment is due Dr. L. L. Thurstone under whose direction and inspiration the method was developed.

some one affective value is associated more frequently with a given situation than is any other value.

For convenience we employ the terminology of Thurstone. He defines the "discriminal process" as that process by which the organism identifies, distinguishes, discriminates, or reacts to stimuli. The fluctuation in the discriminal process for a uniform repeated stimulus is designated the "discriminal dispersion." The discriminal process which is associated more frequently with a given stimulus situation than with any other situation is termed the "modal discriminal process" for that stimulus situation. Thurstone shows that the fundamental psychophysical equation for determining the scale separation between two stimulus situations is

$$S_i - S_j = b_{i-j} \sqrt{\sigma_i^2 + \sigma_j^2 - 2r_{ij}\sigma_i\sigma_j} \quad (1)$$

where

S_i is the modal scale value on the psychological continuum for the stimulus situation i .

S_j is the modal scale value on the psychological continuum for the stimulus situation j .

b_{i-j} is the sigma value as found in a table of the normal probability integral for the experimentally observed proportion of judgments " i less than j ."

σ_i is the standard deviation of the discriminal dispersion of situation i on the psychological continuum.

σ_j is the standard deviation of the discriminal dispersion of situation j on the psychological continuum.

r_{ij} is the correlation between the discriminal deviations for situations i and j .

The assumptions upon which the validity of the above equation rests are given as follows:

(1) Every stimulus in the stimulus-series is associated with a modal discriminal process with which the organism identifies the stimulus for a prescribed attribute.

(2) The modal discriminal process for any given stimulus retains at least some of its identity even when the stimulus is combined with other stimuli into a single perceptual judgment.

(3) The modal processes may be arranged in a linear psychological continuum in the same serial or rank order as the corresponding stimulus series.

(4) In addition to arranging the discriminal processes in rank or serial order, linear separations between them are assigned on the

assumption that the discriminial dispersion for any stimulus is normal on the psychological continuum. This assumption is subject to experimental verification.

The meaning of equation (1) may be indicated diagrammatically by Fig. 2:

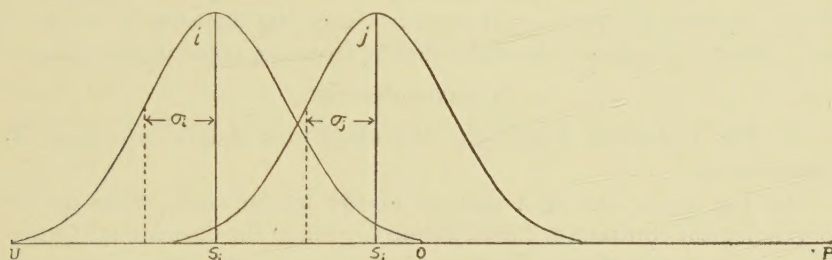


FIG. 2.

The base line of this figure is the affective scale shown in Fig. 1. We have given two situations i and j . For each situation we have a number of different discriminial processes identifying the situation with respect to its affective value. These processes may differ because they come from different individuals, or because they came at different times from the same individual, or both conditions may contribute to the variation. In any event, we obtain a distribution of discriminial processes on the affective scale or continuum for both situation i and j . According to assumption (4) the discriminial dispersions of the two situations respectively are normal. The mean or modal scale value S_i is by definition the affective value of situation i . Similarly S_j is the scale value of situation j . The standard deviations of the discriminial dispersions of the two situations are σ_i and σ_j respectively.

From equation (1) it is clear that the value b_{i-j} is a function both of the distance between S_i and S_j and the dispersions of the two distributions.

If, in addition to the four assumptions previously made we assume that:

(5) The correlation between the discriminial deviations for any two items in the series is zero.

(6) All of the discriminial dispersions in the series are equal. Then equation (1) simplifies to

$$S_i - S_j = b_{i-j}\sigma\sqrt{2} \quad (2)$$

Thus far no unit has been indicated for measuring the distances on

the affective scale. If we adopt σ , or the standard deviation of the discriminial dispersion, as the unit of measurement then equation (2) becomes

$$S_i - S_j = b_{i-j}\sqrt{2} \quad (3)$$

It should be noted, however, that equation (3) gives only the distance between S_i and S_j . It does not give the distance of either S_i or S_j from the origin of the scale, that is, from the point of zero affect.

II. THEORETICAL

1. *The Difference Equation.*—We shall now derive equation (3) analytically.

In Fig. 2, S_i has been defined as the modal scale value on the psychological continuum for situation i . Since the discriminial dispersion is, by assumption, normal on the continuum, the equation to the distribution curve is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-s_i)^2}{2}} \quad (4)$$

where

x is the affective value corresponding to a given discriminial process measured from the point of zero affect in units of the discriminial dispersion.

S_i is the modal scale value measured from the point of zero affect in units of the standard deviation of the discriminial dispersion.

Analytically, S_i is that value which in the equation

$$P = \int_{-\infty}^{x-S_i} f(x) dx \quad (5)$$

makes $P = 0.5$.

Since equation (5) is merely the expression for the proportion of the area of the curve below the ordinate erected at x , it is obvious that $P = 0.5$ when $x = S_i$.

Suppose we represent the distribution of discriminial processes associated with situation i by Fig. 3. Then the probability that a discriminial process corresponding to a value below x will be associated with situation i is given by the proportion of the area below the ordinate at x . If this proportion is given we can substitute it in equation (5) and find the corresponding value of $(x - S_i)$ from a normal probability table. Designating this value by (d) we have

$$x - d = S_i \quad (6)$$

where both x and d are given. Hence S_i can be determined.

Theoretically, then, we might determine the affective value of stimulus situation i by selecting any arbitrary value of x and obtaining from a large group of individuals the proportion of the group which judges the affective value of situation i to be less than x . Substituting this proportion in equation (5), (d) in (6) could be determined and S_i solved for.

Obviously, however, the procedure is not feasible since it would be impossible for individuals to agree on the meaning, in terms of affective value, of any given numerical value. Therefore, instead of dealing with distributions of absolute processes, we deal with distributions of

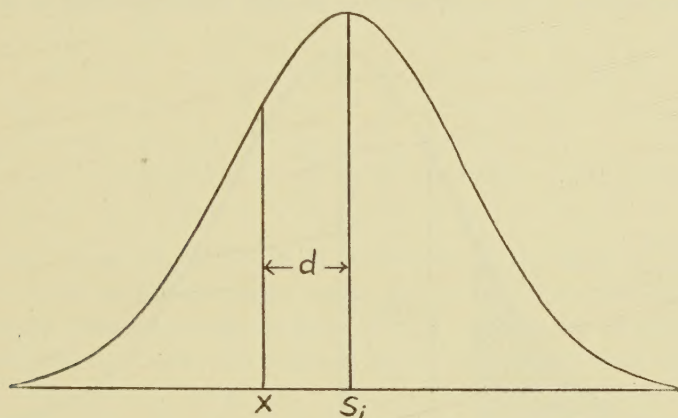


FIG. 3.

differential processes. By an "absolute process" we mean a discriminial process which identifies a single stimulus, whereas, the term "differential" is used to designate a discriminial process which differentiates between two stimuli.

We let x_i and x_j represent the corresponding affective values of the absolute discriminial processes by which a given individual identifies situations i and j , and x_{i-j} the affective value corresponding to the differential process by which he differentiates them.

If the distributions of x_i and x_j are normal, then the distribution of x_{i-j} is also normal. Hence, instead of representing two separate distributions of absolute processes on the psychological continuum, as in Fig. 2, we postulate a single distribution of differential discriminial processes. This is represented diagrammatically in Fig. 4.

Now $S_i - S_j$ is defined as that value for which the probability is 0.5 that a differential discriminial process associated with situations i

and j will lie below $S_i - S_j$. Similarly, the probability is also 0.5 that the process will lie above $S_i - S_j$. Hence $S_i - S_j$ is that value which makes $P = 0.5$ in the equation

$$P_{i-j} = \int_{-\infty}^{x_{i-j} - (S_i - S_j)} f(x_{i-j}) dx_{i-j} \quad (7)$$

In general, if we have given the probability that a differential process associated with situations i and j will lie below any value x_{i-j} , we can find the distance between the value x_{i-j} and $S_i - S_j$. The probability that a differential process corresponding to a value

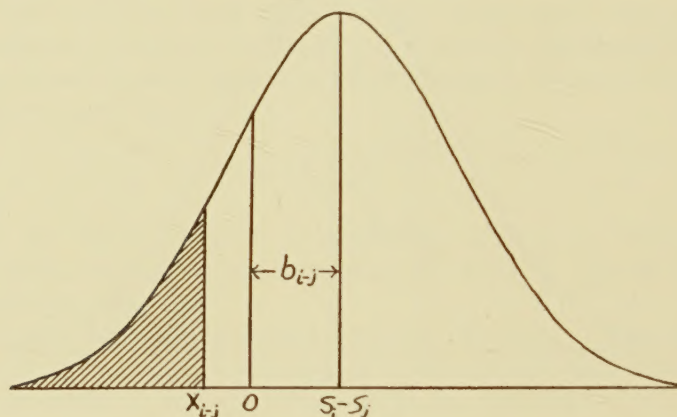


FIG. 4.

below x_{i-j} will be associated simultaneously with situations i and j is given by the proportion of the area of the curve below the ordinate at x_{i-j} . This area is represented by the cross-hatched portion of Fig. 4. If this proportion is given we can substitute it in equation (7) and find the corresponding value of $x_{i-j} - (S_i - S_j)$ from a normal probability table. Designating this value by b_{i-j} , we get

$$x_{i-j} - (S_i - S_j) = b_{i-j}\sqrt{2} \quad (8)$$

where the value $\sqrt{2}$ is the standard deviation of the distribution of differential processes in units of the discriminial dispersion of a distribution of absolute processes.

If we obtain from a large group of individuals the proportion of the group who judge the affective value of situation i to be less than

situation j this means that they judge $S_i - S_j$ to be less than zero. In this case $x_{i-j} = 0$. Substituting this value of x_{i-j} in equation (8) and remembering that b_{i-j} is negative when $S_i < S_j$ and positive when $S_i > S_j$ we get

$$S_i - S_j = b_{i-j}\sqrt{2} \quad (9)$$

which is the same as equation (3). The value b_{i-j} is represented in Fig. 4 by the distance between 0 and the mean $S_i - S_j$.

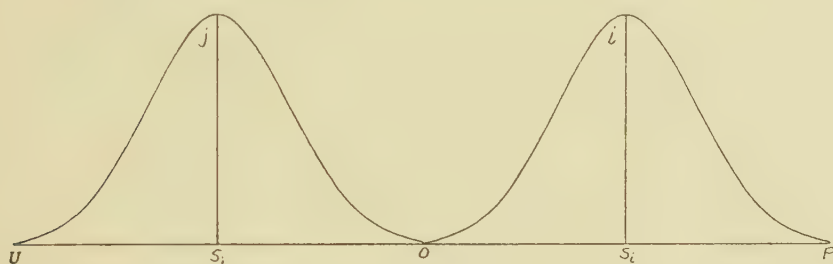


FIG. 5.

2. *The Sum Equation.*—It is evident from the preceding section that in order to find the scale separation between two stimulus situations these situations must be close enough together on the psychological continuum so that one will not be judged greater than the other by all members of the group. The possibility of determining scale separations depends on the fact that there is disagreement as to which of two stimulus situations has the greater value. This principle

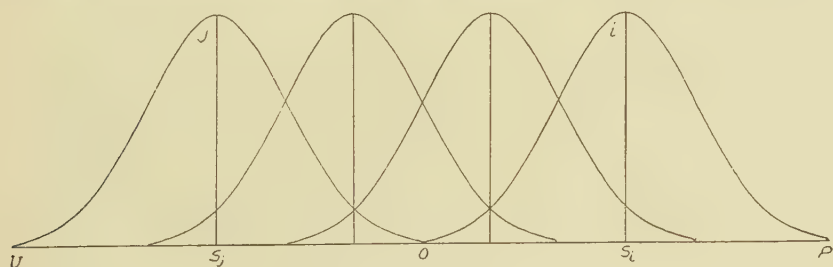


FIG. 6.

has been represented by the overlapping distributions of discriminial processes in Fig. 2.

But suppose we have a situation as indicated in Fig. 5.

Here again we have the same affective scale as in Figs. 1 and 2. The meaning of the distributions is the same as in Fig. 2. But here j

is a very unpleasant situation and i is a very pleasant situation. It is highly improbable that anyone would regard j as a more pleasant situation than i . Since there is not this disagreement, the method of the preceding section cannot be employed to find the distance between S_i and S_j . If, however, we had a number of situations which could be represented by a series of overlapping distributions along the entire range between S_i and S_j , as indicated by Fig. 6, then the method of the preceding section would be available. We could determine the scale distance between the means of each adjacent pair of distributions and by adding all of these obtain the distance between S_i and S_j .

As a matter of fact, this is a part of the procedure actually employed. But, as will be shown later, this procedure if employed alone enables us to obtain only scale separations, whereas we are primarily interested in finding the distance of each scale value from the origin. For this reason, we develop a method for determining the sum of the scale values of two situations.

First we define a "summative process" as that process by which the organism identifies the summative value of two stimulus situations. We let x_{i+j} represent the affective value corresponding to the summative process by which the individual integrates situations i and j .

Retaining the assumptions of the previous section, we can write an equation analogous to (7), namely,

$$P_{i+j} = \int_{-\infty}^{x_{i+j} - (S_i + S_j)} f(x_{i+j}) dx_{i+j} \quad (10)$$

where $S_i + S_j$ is the algebraic sum of the values S_i and S_j , and is defined as that value which makes $P = 0.5$.

To determine this value experimentally, the same logic is employed as in the previous section. We obtain the proportion of individuals who judge the summative value of situations i and j to be less than zero. In equation (10) the value of x_{i+j} in the upper limit of the integral is therefore zero. Since P is given, the upper limit $-(S_i + S_j)$ can be obtained from a normal probability table. Indicating this value by b_{i+j} and multiplying by $\sqrt{2}$, the sigma of the distribution, we get

$$S_i + S_j = b_{i+j}\sqrt{2} \quad (11)$$

remembering that b_{i+j} is negative when $P < 0.5$ and positive when $P > 0.5$.

III. TECHNIQUE

1. *Calculation of Scale Separations by the Average Difference Method.* The method employed by Thurstone¹ in solving for the scale separations of stimulus values is as follows:

Having given the observation equations of the type

$$S_i - S_j = b_{i-j}\sqrt{2}$$

the constant values b_{i-j} are arranged in a skew symmetrical table thus:

	1	2	3	4		n
1	0	$-b_{1-2}$	$-b_{1-3}$	$-b_{1-4}$	$\dots$	$-b_{1-n}$
2	b_{1-2}	0	$-b_{2-3}$	$-b_{2-4}$	$\dots$	$-b_{2-n}$
3	b_{1-3}	b_{2-3}	0	$-b_{3-4}$	$\dots$	$-b_{3-n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	b_{1-n}	b_{2-n}	b_{3-n}	b_{4-n}	$\dots$	0

(12)

where

b_{i-j} is the sigma value of the proportion of individuals who judge $S_i - S_j$ to be less than 0, or, what amounts to the same thing, who judge S_i to be less than S_j .

It will be noted that the entry in the i 'th row of the j 'th column is always the same as the entry in the j 'th row of the i 'th column with sign changed. That is $-b_{ij} = +b_{ji}$. Only the values below the diagonal are given in the observation equations. Those above are copied from those below.

A table of differences is derived from the first table by subtracting column 2 from 1, 3 from 2, 4 from 3, etc. Each remainder in a given difference column is of the type $b_{i(K-1)}$, $-b_{iK}$ so that summing for any given column and multiplying by $\sqrt{2}/n$ we get

$$S_{K-1} - S_K = \frac{\sqrt{2}}{n} \sum (-b_{i-(K-1)} + b_{i-K}) \quad (13)$$

giving the scale separation between S_{K-1} and S_K .

The same general procedure may be followed when the equations are of the type

$$S_i + S_j = b_{i+j}\sqrt{2}$$

In this case, however, the table corresponding to (12) is not skew symmetrical, but perfectly symmetrical. That is,

	1	2	3	4	$\dots$	n
1	—	b_{1+2}	b_{1+3}	b_{1+4}	$\dots$	b_{1+n}
2	b_{1+2}	—	b_{2+3}	b_{2+4}	$\dots$	b_{2+n}
3	b_{1+3}	b_{2+3}	—	b_{3+4}	$\dots$	b_{3+n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	b_{1+n}	b_{2+n}	b_{3+n}	b_{4+n}	$\dots$	—

(14)

This is due to the fact that we are considering sums of scale values rather than differences. Furthermore, instead of having a diagonal of zeros the diagonal elements are indeterminate, and hence left blank.

In forming the difference table of (14) each subsequent column is subtracted from the preceding one, so that each remainder in a given column is of the type $b_{i+(K-1)} - b_{i+K}$. Summing each difference column and multiplying by $\frac{\sqrt{2}}{n-2}$ gives,

$$S_{K-1} - S_K = \frac{\sqrt{2}}{n-2} \sum (b_{i+(K-1)} - b_{i+K}) \quad (15)$$

The set of scale separations obtained from (15) may be averaged with those obtained from (13).

2. *The Least Square Method.*—But while either (13) or (15), or a combination of the two, give scale separations, they give no clue as to the origin of the scale. Since our fundamental problem is the determination of this absolute origin, we must develop a method for combining the sets of equations

$$S_i - S_j = b_{i-j}\sqrt{2} \quad (9)$$

$$S_i + S_j = b_{i+j}\sqrt{2} \quad (11)$$

so that the S 's can be determined in terms of their distance from this absolute origin. To determine the scale values in terms of an absolute origin a "least squares" procedure is employed. The normal equations are derived from the observation equations in the conventional manner, but due to the form of the observation equations the normal equations are much easier to derive than with the ordinary (n)-variable problem. The derivation is as follows:

$$\Sigma(S_i - S_j - b_{i-j}\sqrt{2})^2 = V = \text{a minimum} \quad (16)$$

$$\Sigma(S_i + S_j - b_{i+j}\sqrt{2})^2 = U = \text{a minimum} \quad (17)$$

Considering first equation (16), we have

$$\frac{\partial V}{\partial S_1} = 0, \frac{\partial V}{\partial S_2} = 0, \dots \frac{\partial V}{\partial S_n} = 0$$

The general form for any equation (K) is,

$$\frac{\partial V}{\partial S_K} = -S_1 - S_2 \dots + S_K(n-1) \dots - S_n - \sqrt{2} \left(\sum_{K+1}^n b_{Ki} - \sum_1^{K-1} b_{Ki} \right) = 0 \quad (18)$$

There will be (n) of these equations. First we determine the nature of the matrix of the coefficients (18), leaving the discussion of the formation of the constant terms for a latter paragraph. An examination of (18) shows that the matrix of the coefficients of the parameters or scale values takes the form

$$A_{i-j} = \begin{vmatrix} n-1 & -1 & -1 & \dots & -1 \\ -1 & n-1 & -1 & \dots & -1 \\ -1 & -1 & n-1 & \dots & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -1 & -1 & -1 & \dots & n-1 \end{vmatrix} \quad (19)$$

But since the number of -1's in every row of (19) is (n - 1), the sum of the elements of every row is zero. According to the theorem in the elementary theory of equations that if the sum of the elements of every row is zero, the determinant of the matrix vanishes, A_{i-j} is zero and there is no unique solution for the constants in equation (18). This necessitates a combination with the normal equations derived from (17). The method of combining the two sets of equations will be discussed later.

Recurring now to the formation of the constant term in (18), it will be noted that the general expression for the K'th constant term is

$$K_K = \sqrt{2} \left(\sum_{K+1}^n b_{Ki} - \sum_1^{K-1} b_{Ki} \right) \quad (20)$$

But we see that (20) is precisely the result obtained if we sum the K'th column in table (12) and multiply by $\sqrt{2}$, where the summations take into account that all the b's above the diagonal have a negative

sign. This procedure enables us to get the respective constant terms directly from the table of b 's.

As indicated, however, equations (18) do not give a solution of the parameters, due to the fact that the determinant of the matrix A_{i-j} in (19) vanishes. Now let us consider the normal equations to be derived from (17).

Taking partial derivatives with respect to each of the S 's and equating to zero gives (n) equations. These are of the form

$$\frac{\partial U}{\partial S_K} = S_1 + S_2 \cdot \cdot \cdot + S_K(n-1) \cdot \cdot \cdot + S_n - \sqrt{2}(\Sigma b_{Ki}) = 0 \quad (21)$$

As in (18) the constant terms of these equations are formed by summing the columns in the table of b_{i+j} 's and multiplying by $\sqrt{2}$.

We see that while the constant terms for (18) are given by

$$K_{i-j} = \sqrt{2} \left(\sum_{K+1}^n b_{Kj} - \sum b_{Ki} \right) \quad (20)$$

the constant terms for (21) are given simply by

$$K_{i+j} = \sqrt{2}(\Sigma b_{Ki}) \quad (22)$$

The reason, of course, that we have a difference of two summations in (20) and not in (22) is due to the fact that while in table (12) all the b 's above the diagonal are given the negative sign, in table (14) they retain the positive sign. That is, the table is perfectly symmetrical. The method of obtaining the constant terms by summing rows is identical in both cases.

Now let us consider the matrix of the coefficients of (21). An examination of these equations shows that the matrix is given by

$$A_{i+j} = \begin{vmatrix} n-1 & +1 & +1 & \cdot \cdot \cdot & +1 \\ +1 & n-1 & +1 & \cdot \cdot \cdot & +1 \\ +1 & +1 & n-1 & \cdot \cdot \cdot & +1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ +1 & +1 & +1 & \cdot \cdot \cdot & n-1 \end{vmatrix} \quad (23)$$

A comparison of (19) and (23) shows that they differ only in that the 1's in (19) are negative, while in (23) they are positive. In both matrices the diagonal terms are the same, viz., $(n-1)$.

Now the determinant of (23) does not vanish, so that equations (21) determine uniquely the values of the S 's. However, in an experi-

mental set-up it is usually not feasible to get a matrix as indicated in (23). This is due to the fact that for some situations the proportion of judgments $S_i + S_j > 0$ is close to either zero or unity and hence too unreliable to use. For the same reason the matrix (19) obtained from experimental data differs also from that given. In an experimental set-up many of the elements which are -1 and 1 in (19) and (23) respectively are zero. When this is true the diagonal terms will not be $(n - 1)$, but for any row the diagonal term will be equal to the number of 1's in that row. If it were possible to obtain complete equations of the types (18) and (21) respectively, then the K 'th equation of (18) could be added to the K 'th equation of (21) giving simply

$$S_K = \frac{\sqrt{2}}{2(n-1)} \left[\left(\sum_{K+1}^n b_{K-i} - \sum_1^{K-1} b_{K-i} \right) + \sum_1^n b_{K+i} \right] \quad (24)$$

so that S_K could be obtained directly by adding the summation of the K 'th column in (12) to the summation of the K 'th column in (17) and multiplying this sum by $\frac{\sqrt{2}}{2(n-1)}$. If we have $\frac{n(n-1)}{2}$ observation equations of the type $S_i - S_j = b_{i-j}\sqrt{2}$ and the same number of the type $S_i + S_j = b_{i+j}\sqrt{2}$ then (24) is the expression which makes $\sum (\bar{S}_i \pm \bar{S}_j - b_{i \pm j}\sqrt{2})^2$ a minimum where the sign $\pm$ indicates that the summation is through both the $S_i - S_j$ and the $S_i + S_j$ series.

As a matter of fact, in an experimental set-up for equations of the type $S_i + S_j = b_{i+j}\sqrt{2}$ the b 's can be obtained only for $S_i + S_j$ when

$$S_i > 0 > S_j$$

or

$$S_i < 0 < S_j \quad (25)$$

The inequalities (25) may not be required providing the ratio S_i/σ_i is comparatively small. If these do not obtain, then (23) takes the form

$$A_{i+j} = \begin{vmatrix} a_1 & . & 0 & 1 & . & 1 \\ . & . & . & . & . & . \\ 0 & . & a_m & 1 & . & 1 \\ 1 & . & 1 & a_{m+1} & . & 0 \\ . & . & . & . & . & . \\ 1 & . & 1 & 0 & . & a_n \end{vmatrix} \quad (26)$$

where the S 's corresponding to all terms up to (m) are negative and those from $(m + 1)$ to (n) are positive, but where the a 's are still equal to the sum of the 1's in their respective rows.

Now (26) vanishes, so that if the experimental data result in a determinant of this type there is no solution for the S 's in (21).

We next note that when the matrices obtained from the experimental data vary from (19) and (23) respectively this variation is due to the fact that certain of the b 's in tables (12) and (14) are missing. But knowing which of these b 's are missing the matrices can always be set up directly from the tables.

But to determine the S 's uniquely, equations (18) and (21) are added together yielding a matrix which is the sum of (19) and (23).

The normal equations may be set up directly from tables (12) and (14) as follows:

To get the diagonal element for any row, count the total number of entries for that row in the corresponding rows of both tables. This number will be the diagonal element. To get any other element:

(1) If its corresponding row and column has an entry for both (12) and (14) the element is zero.

(2) If its corresponding row and column lacks an entry in both (12) and (14) the element is zero.

(3) If its corresponding row and column has an entry in only table (12), the element is -1 .

(4) If its corresponding row and column has an entry in only table (14), the element is $+1$.

In a numerical set up, only those terms above the diagonal need be written down.

The constant term for any row is obtained by adding the sum of the elements in the corresponding column in table (12) to the sum of the elements in the corresponding column of table (14) and multiplying this total by $\sqrt{2}$.

The equations may be solved most conveniently by the Doolittle method (4). The solution is considerably simplified since all of the coefficients except the diagonal terms are either plus or minus 1, or zero.

3. *Correspondence of Calculated with Observed Values.*—To find the degree of accuracy with which the solutions fit the experimental data, we determine the average squared deviation in the equations

$$(\overline{S_i} - \overline{S_i} - b_{i-i}\sqrt{2})^2 = d^2_{i-i}$$

and

$$(S_i + S_j - b_{i-j}\sqrt{2})^2 = d^2_{i+j}$$

where the values obtained for the S 's are substituted in the equations.

The value d_{i-j} is the discrepancy obtained by subtracting the observed difference between the scale values of two stimulus situations from the calculated difference. The value d_{i+j} has the same meaning for the sum of two scale values. The square root of the average of all of these deviations is called the standard error of estimate and is written

$$\sigma_{\text{est.}} = \sqrt{\frac{\sum d^2}{n}}$$

where (n) is the total number of both sum and difference equations. Of course, $\sigma_{\text{est.}}$ may be reduced to $\text{PE}_{\text{est.}}$ by the relation

$$\text{PE}_{\text{est.}} = 0.6745\sigma_{\text{est.}}$$

Now in any regression equation of the general form

$$Y = B_1x_1 + B_2x_2 + \dots + B_nx_n$$

the $\text{PE}_{\text{est.}}$ means that approximately fifty per cent of the experimental values will not deviate from the corresponding calculated values by more than the numerical value of the $\text{PE}_{\text{est.}}$. It may be interpreted as a measure of the error in the prediction or regression equation.

But in our problem the regression equation takes the simpler form

$$S_i - S_j = b_{i-j}\sqrt{2}$$

or

$$S_i + S_j = b_{i+j}\sqrt{2}$$

Here, then, the PE is a measure of the error in a sum or difference of two values. But the error of a sum or difference of two values is equal to the square root of the sum of the squares of their respective errors.

Hence, if $\text{PE}_{\text{est.}} \equiv \text{PE}_{ij}$ is the error in $S_i - S_j$ or $S_i + S_j$ and PE is the error in any given scale value, then

$$\text{PE}_{ij} = \sqrt{(\text{PE}_i)^2 + (\text{PE}_j)^2} \quad (27)$$

If we assume that the errors in all the scale values are equal we obtain from (27)

$$\text{PE}_i = \frac{\text{PE}_{ij}}{\sqrt{2}} \quad (28)$$

To get PE_{ij} we need not actually find the residual for each equation and then square and sum. From the method of fitting it follows that for any "least square" solution of the type

$$\Sigma(B_1x_i + B_2x_2 + \dots + B_nx_n - x_0)^2 = \text{a minimum,}$$

that the standard error of prediction (5) is given by,

$$\sigma_{est.} = \left[\frac{\Sigma x_0^2 - (B_1x_0x_1 + B_2x_0x_2 + \dots + B_nx_0x_n)}{n} \right]^{1/2} \quad (29)$$

It will be noted that Σx_0^2 is the sum of the squares of the dependent variable. In our problem this is

$$\Sigma x_0^2 = 2\Sigma b^2_{ij} \quad (30)$$

where the sign $\pm$ indicates summation through both the difference and sum series. Furthermore,

$$B_1 = S_1, B_2 = S_2, \dots B_n = S_n \quad (31)$$

and

$$\Sigma x_0x_1 = K_1, \Sigma x_0x_2 = K_2, \dots \Sigma x_0x_n = K_n \quad (32)$$

Substituting values (30), (31), and (32) in (29), we have

$$\sigma_{est.} = \left[\frac{2\Sigma b^2_{ij} - \Sigma S_i K_i}{n} \right]^{1/2} \quad (33)$$

or from (28)

$$PE_i = 0.6745 \sqrt{\frac{2\Sigma b^2_{ij} - \Sigma S_i K_i}{2n}} \quad (34)$$

Equation (32) is the expression for the PE of a scale value.

To obtain the value Σb^2_{ij} , tables are made up from tables (12) and (14) by squaring the values in these tables. Only the sections above the diagonals are included in the tables of squares, as those below have the same value when squared. The sums of the rows of both tables are added together to get Σb^2_{ij} .

The value $\Sigma S_i K_i$ is readily obtained by multiplying each constant term by the corresponding scale value and summing.

IV. EXPERIMENTAL

1. *Method of Obtaining Values for the Difference and Sum Equations.*
The equations to be kept in mind in an experimental set-up are

$$P_{i-j} = \int_{-\infty}^{x_{i-j}-(S_i-S_j)} f(x_{i-j}) dx_{i-j} \quad (7)$$

with its solution for $S_i - S_j$,

$$S_i - S_j = b_{i-j}\sqrt{2} \quad (9)$$

and

$$P_{i+j} = \int_{-\infty}^{x_{i+j}-(S_i+S_j)} f(x_{i+j}) dx_{i+j} \quad (10)$$

with its solution

$$S_i + S_j = b_{i+j}\sqrt{2} \quad (11)$$

The method for determining P in (7) is as follows:

A series of stimulus situations is given ranging in affective value from distinctly positive to distinctly negative. Denoting the situa-

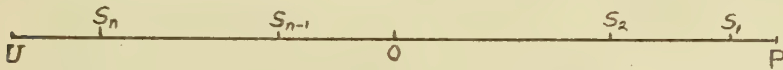


FIG. 7.

tions by S_1, S_2 to S_n , we may represent them diagrammatically on a scale as in Fig. 7:

Now when we say that situation 1 is more pleasant than situation 2 for a given individual, we mean that he prefers 1 to 2. By definition, 1 has for him a higher affective value than 2. Hence, to find whether situation i is greater in affect than situation j , statements of the situations are assembled according to the method of paired comparisons on mimeographed sheets with phrasing as follows:

- | | |
|----------------------|--|
| () 1-2. | I would rather have S_1 than S_2 . |
| () 1-3. | I would rather have S_1 than S_3 . |
| | |
| () 1- n . | I would rather have S_1 than S_n . |
| () 2-3. | I would rather have S_2 than S_3 . |
| | |
| () 2- n . | I would rather have S_2 than S_n . |
| | |
| () ($n-1$)- n . | I would rather have S_{n-1} than S_n . |

These are presented to a group of individuals. If the individual considers the statement true for himself, he marks (+) in the parenthesis. If not, he marks (-), just as in a True-False test. He is

instructed to leave no blanks. For each statement of paired situations, then, there is a certain proportion of individuals who marked it (+). This is the value of P in (7) and from which the value b_{i-j} in (9) is obtained from a table of the normal probability integral. These b_{i-j} values are the ones entered in table (12).

When we say that situation 1 is more pleasant for a given individual than (n) is unpleasant, we mean that if he were permitted to enjoy 1, he would be willing to submit to (n). By definition, the cumulative affect of the two situations is, for him, positive. Hence, to find how much the cumulative affect of situations i and j differ from zero, the situations are assembled on a second set of mimeographed sheets with phrasing as follows:

- | | |
|----------------------|--|
| () 1-2. | If I could have S_1 I would be willing to take S_2 . |
| () 1-3. | If I could have S_1 I would be willing to take S_3 . |
| () 1- n . | If I could have S_1 I would be willing to take S_n . |
| () 2-3. | If I could have S_2 I would be willing to take S_3 . |
| () 2- n . | If I could have S_2 I would be willing to take S_n . |
| () ($n-1$)- n . | If I could have S_{n-1} I would be willing to take S_n . |

These are presented to the same group of individuals as was the first series. Again for each statement of paired situations, the proportion of individuals who marked it (+) is obtained. This percentage is the value of P_{i+j} in (10) and from which the value b_{i+j} in (11) is obtained from a normal probability table. These b_{i+j} values are the ones entered in table (14).

On the mimeographed forms for the first series, however, pairs of situations which are obviously far apart on the scale may be omitted, for they would result in proportions either too high or low to be reliable. For the same reason in the second series, situations which are obviously close together, and clearly some distance from the point of zero desirability or affect, are also omitted from the series.

In the experimental series the situations were obtained from a number of the individuals who judged the pairs, as follows:

The subjects were one hundred sixty students in a teachers college. The following instructions were given:

Describe in one sentence a situation which you consider extremely unpleasant, one which you consider extremely pleasant, one which you consider as being

entirely neutral, another which is moderately unpleasant, and still another which is moderately pleasant.

Thus each individual submitted five situations. In all, about two hundred situations were obtained. The experimenter then selected twelve out of these two hundred according to two criteria:

(1) Subjectively they appeared to be relatively uniformly extended through a scale from extreme unpleasantness to extreme pleasantness.

(2) When possible, situations were selected which were most frequently described.

The object of (2) was, of course, to select situations which were relevant to the everyday experience of the group which was to judge them. They were then edited so as to fit as logically and grammatically as possible into the form,

I would rather have S_i than S_j ,
and

I would be willing to take S_i if I could have S_j , respectively.

The situations which were put in the mimeographed forms were phrased as follows:

1. Receive a kiss from the one I love.
2. Dance with a good dancer.
3. Go to a good musical comedy.
4. See a beautiful art exhibit.
5. Receive a smile from a friend of the opposite sex.
6. Watch a strange man try to choose between a red and a green necktie.
7. Lose several hairs from my head.
8. Hear a beginner practise his violin lesson.
9. Discover I had forgotten my purse after having purchased an article.
10. Have my clothes splashed by a passing vehicle when going to a party.
11. See my sweetheart come into the show with another date.
12. Spend a night in jail.

2. *Data and Calculations.*—Table I gives the (b) values for both the sum and the difference series. Table II shows the normal equations set up from Table I in the manner previously outlined, and ready for solution. The scale values, as determined from a solution of these equations is given in Table III.

To find the PE of a scale value, we employ the formula

$$PE = 0.6745 \sqrt{\frac{2 \sum b^2_{ij} - \sum S_i K_i}{2n}} \quad (32)$$

To get Σb^2_{ij} , another table is constructed from Table I by squaring only those b 's above the diagonal. Multiplying the sum of all the b^2 's by 2, we get,

$$2\Sigma b^2_{ij} = 150.6480$$

The value $\Sigma S_i K_i$ is obtained by multiplying column K of Table II by the column of scale values in Table III and summing the products. We find that

$$\Sigma S_i K_i = 145.5676$$

The number of observation equations is

$$N = 56.$$

Substituting in the formula

$$PE = 0.6745 \sqrt{\frac{150.6480 - 145.5676}{112}} = 0.1442$$

The PE of a scale value, then, is 0.1442.

TABLE I.—SIGMA VALUES OF PROPORTIONS

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	Σ
Series 2													
1									-.34	-.78	-.99	-1.51	-3.62
2								.47	-.61	-1.19	-.98	-2.01	-4.32
3								1.39	-.21	-.94	-.79	-1.96	-2.51
4							-.45	.87	-.61	-1.10	-1.08		-1.47
5							-.45	-.05	-1.23	-1.55	-1.67		-4.95
6							-.91	-.70	-1.77				-3.38
7				.45	-.45	-.91							
8		.47	1.39	.87	-.05	-.70							
9	-.34	-.61	-.21	-.61	-1.23	-1.77							
10	-.78	-1.19	-.94	-1.10	-1.55								
11	-.99	-.98	-.79	-1.08	-1.67								
12	-1.51	-2.01	-1.96										
Series 1													
1		-.58	-.04	-.39									
2		.58		.38	.06	-.66							
3		.04	-.38		-.50	-1.26	-1.57						
4		.39	-.06	.50		-.85	-1.38	-1.85					
5			.66	1.26	.85		-1.07	-1.85					
6				1.57	1.38	1.07		-1.63	-1.28	-1.70			
7					1.85	1.85	1.63		.44	-.63	-1.17		
8							1.28	-.44		-1.35	-1.90	-1.54	
9						1.70	.63	1.35		-.58	-.77	-2.00	
10							1.17	1.90	.58		-.42	-1.63	
11								1.54	.77	.42		1.11	
12									2.00	1.63	1.11		
Total	-2.61	-4.68	1.16	1.78	-4.80	-2.79	-4.88	5.93	-5.10	-7.16	-7.13	-10.22	

TABLE II

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	K	Σ
1	7.0000	-1.000	-1.000	-1.000					1.000	1.000	1.000	1.000	3.6911	11.691
2		9.000	-1.000	-1.000	-1.000			1.000	1.000	1.000	1.000	1.000	6.6185	16.618
3			10.000	-1.000	-1.000	-1.000		1.000	1.000	1.000	1.000	1.000	-1.6405	8.359
4				11.000	-1.000	-1.000		1.000	1.000	1.000	1.000		-2.5173	7.483
5					10.000	-1.000		1.000	1.000	1.000	1.000		6.7882	16.788
6						9.000							3.9456	9.946
7							9.000	-1.000	-1.000	-1.000			6.9013	12.901
8							0	10.000	-1.000	-1.000	-1.000		-8.3862	1.6138
9									12.000	-1.000	-1.000	-1.000	7.2124	19.212
10										10.000	-1.000	-1.000	10.1257	20.126
11											9.000	-1.000	10.0832	20.083
12												6.000	14.4531	20.453

TABLE III

SITUATION	ABSOLUTE SCALE VALUES
3	2.49
1	2.48
4	1.93
2	1.90
5	1.03
6	0.17
8	-1.01
7	-1.55
9	-2.64
10	-3.39
11	-3.57
12	-5.15

V. SUMMARY

1. The scaling method developed by Thurstone is described, and the assumptions underlying the method are presented.

2. The equations required in solving for the absolute affective values of a series of stimulus situations are derived analytically. The derivations include the assumptions involved in the method of scaling.

3. Two methods for determining scale values are presented:

(a) The "average difference" method yields only scale separations and necessitates the arbitrary selection of an origin.

(b) The "least square" method gives a solution of the affective values of stimulus situations in terms of their distances from an absolute zero.

4. The normal equations for the least square solution are derived and their typical form discussed.

5. Instructions are outlined for writing the normal equations directly from the table of sigma values of the experimental proportions.

6. Statistical procedure is presented for determining how accurately the calculated scale values fit the experimental data.

7. An experimental method is devised for obtaining data in the form required for the scaling equations.

8. To demonstrate the scaling method a model experiment was conducted; the data resulting from the experiment are analyzed according to the method outlined and the results of the calculations are presented.

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